

Revealing A Possible Implication By Imposing Lee-Yang Theorem On The Partition Function of The Universe

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The goals of this note are twofold: First, to revisit a mechanism proposed by Hořava and Minic[1] which generates a zero cosmological constant to the Universe based on Boltzmann probability distribution and the Holographic principle, a comparison between the zero cosmological constant and recent observation results is given. Secondly, in order to investigate the possibility of phase-transition phenomena of the Universe, a further study on an exponential class of the partition function of the Universe which is given by Shaw and Barrow [5, 6] was provided. The contribution of this paper is that after applying the fundamental theorem of algebra to one action chosen from the exponential class that derived from the Shaw and Barrow's works, the Lee-Yang theorem can be applied to the selected action, meaning that critical phenomena might involve the growth of large-scale structure of the Universe, and the phase transition phenomena of the Universe might exist during the spacetime expands.

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I. INTRODUCTION

Cosmological constant(CC) problem is becoming one of the most important puzzles of theoretical physics[7–12]. In 1917, CC was first introduced by Einstein in order to allow a static cosmological solution when General Relativity(GR) is applied. Based on some strong evidences from today's astrophysical observation one can confidently say that CC is nonzero, but small and positive for our universe. Additionally, one might have already known that there are several possible candidates for cosmological constant; for instance, the method of summing over all contributions from variety quantum vacua energy with all possible sources, e.g. quantum chromodynamics transitions, zero-point energy and Casimir energy in quantum electrodynamics. Still and all, there is a gap between the theoretical and the observational works of CC in 54 order of magnitude. Thus, one of the greatest conundrums in physics is to answer the question about why CC is nonzero? And then needed to explain why CC is small and positive? Explaining today's tiny value requires a mechanism which could be capable on canceling many different contributions with near-perfect precision.

II. A SUMMARY OF DERIVATION AND RESULTS IN [1]

In conformity with [1], Hořava and Minic pointed out that for a large class of universes, in combination with certain robust thermodynamic arguments, cosmic holo-

graphy strongly forced that the most probable value for the cosmological constant is zero. Additionally, in four spacetime dimensions, the probability distribution takes the Baum-Hawking form, $dP \approx \exp(cM_p^2/\Lambda)d\Lambda$. Because it has been suggested on general grounds[13–15] that cosmic holography[16–19] ought to be relevant to solving the cosmological constant problem. A summary of the argument is that the cosmological constant problem in local quantum field theory is a naturalness problem, following from a gross overcount of the degrees of freedom of the vacuum in the bulk.

According to [1], the probability distribution for the CC to have a value in the interval from Λ to $\Lambda + d\Lambda$ is taken in a Boltzmann probability distribution

$$\omega(\Lambda)d\Lambda = \text{const} \exp\{S(\Lambda)\}, \quad (1)$$

where the Boltzmann constant is set to one. $S(\Lambda)$ is the holographic entropy, and its derivation can be found in [1]. Then one might conclude that

$$\omega(\Lambda)d\Lambda = \text{const} \exp\left\{\frac{cM_p^2}{\Lambda}\right\}. \quad (2)$$

Here c stands for a constant of order-one which takes into account the neglected numerical factors. This formula implies that the probability distribution is strongly peaked around the value $\Lambda = 0+$. Hence, in [1], Hořava and Minic concluded that the most probable value of the CC, as implied by holography and thermodynamics, is zero.

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III. A COMPARISON BETWEEN THE THEORETICAL AND EXPERIMENTAL WORKS

In 1917, the Cosmological constant(CC), Λ , was proposed by Einstein. The vital role CC played is to provide the gravitational repulsion needed to support a static universe. It was not until 1998 that there were two independent groups, led by Riess[2] and Perlmutter[3] used Type 1a supernovae to show that the universe is accelerating. This discovery provided a first evidence that Λ is nonzero, instead of zero value, $\Lambda \approx 1.7 \times 10^{-121}$ Planck units. Before their discovery, many particle physicists suspected that some fundamental principles must force its value to be zero, but now in conformity with observation, one knows that this is not the case. In other words, there must something wrong in [1]. Thus, beside this observational reason, author would built up a theoretical reason as well. One of the main goals in this note is trying to find the possible loophole in [1], and then find some possible way-outs.

IV. DISCUSSIONS

A. Issues of Zero Cosmological Constant Generating Mechanism

The first issue is due to the equation (2), it demands that Λ ought to be zero. After Riess[2] and Perlmutter[3] proposed their *de facto* discovery, the notion of nonzero CC became well-known. In 1984, this mathematical form had been first proposed by S. Hawking[21] by using his Euclidean action approach of quantum gravity. Followed with Hawking's approach, in 1988, S. Coleman gave some arguments of this form when he discuss the effect of the wormholes with wavefunction of the universe [22, 23]; later on, Y. J. Ng and H. van Dam also gave some arguments[24, 25]. They argued that the distribution of values should be peaked about $\Lambda = 0$ (i.e. $f_{prior}(\Lambda) = \exp(3\pi / G\Lambda)$, where $f_{prior}(\Lambda)$ is the prior probability distribution, a parallel but different approach can be found in Weinberg(1987)[26]).

The second issue is about finding an actual microscopic theory that satisfies holography. In equation (1) and (2), the assumption has some ambiguities, and in essence, here is a distinction between Hawking-Coleman's approach[21–23] and Hořava-Minic[1]. In Hawking-Coleman, their assumptions are based on the understanding of the Euclidean action, and then they write down the partition (wave) function of the Universe. With a different perspective, Hořava-Minic's approach is based on two assumptions which are thermodynamics and cosmic holography. The problem is that suppose quantum theory is a right direction to pursue with, then a microscopic perspective and precise description is necessary.

B. A Possible Way-Out for the First Problem

For concreteness, consider the following gravitational field equation:

$$G^{\mu\nu} = \kappa \langle T^{\mu\nu} \rangle, \quad (3)$$

where $\langle T^{\mu\nu} \rangle$ denotes the expected value of the energy-momentum tensor of matter in the whole Universe, $\kappa = 8\pi G, c = \hbar = 1$, $G^{\mu\nu}$ is the Einstein tensor which is with the following expression: $G^{\mu\nu} = R^{\mu\nu} - g^{\mu\nu} R/2$, and $R^{\mu\nu}$ is Riemann tensor, i.e., so-called the Ricci curvature of $g^{\mu\nu}$ and R is Ricci scalar.

Now, let's introduce a bare CC, λ , required adding a term $-\lambda g^{\mu\nu}$ to the original gravitational field equation:

$$G^{\mu\nu} = \kappa \langle T^{\mu\nu} \rangle - \lambda g^{\mu\nu}. \quad (4)$$

The next step is taken quantum fluctuation in to account. Quantum fluctuation results in vacuum energy, ρ_{vac} , which will contribute to $\langle T^{\mu\nu} \rangle$.

$$\langle T^{\mu\nu} \rangle = T_m^{\mu\nu} - \Lambda g^{\mu\nu}, \quad (5)$$

where $\Lambda = \lambda + \kappa \rho_{vac}$. Thus, the vacuum energy, $\kappa \rho_{vac}$, contributes to the effective CC, Λ . At the late cosmic times, ρ_{vac} does not change. In conformity with the standard model of the particle physics, $\rho_{vac}^{eff} \approx \kappa^{-1} \Lambda \gtrsim M_{EW}^4$, where M_{EW}^4 denotes the electroweak energy density, $M_{EW}^4 \approx (246 \text{ GeV})^4$. Even so, this is contradict to the current measurement in the observational astrophysics[4] that $\rho_{vac}^{eff} \approx (2.4 \times 10^{-12} \text{ GeV})^4$. Thus, there is a 54 order gap between observation and the theory which is based on a semi-classical combination of GR and the standard model of particle physics. This conundrum of CC is conceptually mentioned in section I that which also usually be called (old) CC problem. To see more deeper, let's focus on cosmic time, which is corresponding to effective CC, t_Λ . t_Λ can be derived from effective CC as follows:

$$t_\Lambda \approx \Lambda^{-1/2} \approx 9.7 \text{ Gyr}, \quad (6)$$

which is pretty close to the present age of the Universe $t_U \approx 13.7 \text{ Gyr}$. This is usually called new CC problem or coincidence problem. To be more precisely, the problem is why the fixed time t_Λ ought to be correlated to the observer-dependent time t_U ? Another perspective of coincidence problem is to usually pay attention on the problem that at present day why matter density is close to the dark energy density, if one takes vacuum energy as a candidate to dark energy then the two perspectives will merge into one.

So far there is one interesting possible way-out of the first problem in the previous subsection[5, 6] and in the meanwhile this way-out can provide a resolution of old and new CC problems. Basically, they proposed a simple extension of the usual action principle in which the effective CC, Λ , will be promoted from a parameter to a "field". The variation leads to a new field equation which determine the value of λ , and thus the effective

CC, in terms of other properties of the observed Universe. The point is, one can derive the observed classical history naturally has $t_\Lambda \sim t_U$. Crucially, when this approach applied to GR, λ (and therefore, Λ except when ρ_{van} evolves owing to, say, a phase transition) is a true constant and is not seen to evolve. Thus, the resulting history is indistinguishable from GR with the value of put in by hand. Withal, their theory can be tested in the near future by Planck-CMB data.

For the *de facto* reason, let's define an action, $I_{tot}[\mathbf{g}_{\mu\nu}, \Psi^a, \Lambda; \mathcal{M}]$, of the whole Universe on a manifold \mathcal{M} , with boundary $\partial\mathcal{M}$, and effective CC, Λ , matter fields Ψ^a and metric $\mathbf{g}_{\mu\nu}$. Usually, bare CC, λ , is a fixed parameter and the wave (partition) function of the Universe, $Z[\lambda; \mathcal{M}] \equiv Z_\Lambda[\mathcal{M}]$, is proffered by

$$Z_\Lambda[\mathcal{M}] = \sum e^{iI_{tot}} \times [\text{gauge fixing terms}], \quad (7)$$

where $\{Q^a\}$ are some fixed boundary quantities which are generalized charges on $\partial\mathcal{M}$, and the sum is over all histories (i.e., configurations of the metric and matter, $\mathbf{g}_{\mu\nu}, \Psi^a$) is consistent with these fixed charges. The dominant contribution to $Z_\Lambda[\mathcal{M}]$ is from the histories for which I_{tot} is stationary for $\mathbf{g}_{\mu\nu}$ and Ψ^a variations that preserve the $\{Q^a\}$. In these dominant histories, the matter and metric fields obey their classical field equations. Their proposal for solving the CC problems is by promoting the bare CC, from a fixed parameter to a field. This procedure can be rise in a fundamental theory, e.g., string theory, where there are a large number of string vacua, so-called landscape, with different minima of the vacuum energy. Further detail can be found in [5, 6].

C. A Further Discussion For The First Possible Way-Out

Because Shaw-Barrow's theory can be tested in the near future, thus it is worthwhile to make a more fur-

ther discussion and analysis. In conformity with [5, 6], one can simply thinking in an *ad hoc* solution which is a homogeneous and isotropic cosmological metric:

$$ds^2 = a^2(\tau)[-d\tau^2 + (1 + kx^2/4)^{-2}dx^i dx^i], \quad (8)$$

where k denotes the spatial curvature. The observer is at $(\tau, x) = (\tau_0, 0)$ and $\partial\mathcal{M}$ is at the surface $\tau = 0$ where $a = 0$. Take

$$T_m^{\mu\nu} = (\rho_m + P_m)U^\mu U^\nu + P_m g^{\mu\nu}, \quad (9)$$

where $U^\mu = -a^{-1}(\tau)\nabla^\mu \tau$. With

$$H = \frac{a, \tau}{a^2}, \quad (10)$$

Einstein's equations tender

$$H^2 = \frac{\kappa\rho_m}{3} + \frac{\Lambda}{3} - \frac{k}{a^2}, \quad (11)$$

and

$$\frac{\rho_{m, \tau}}{a} = -3H(\rho_m + P_m). \quad (12)$$

In [5], Shaw and Barrow found that up to the linear order $O(kx^2)$, the action can be rewritten as follows:

$$I_{cl} = \frac{4\pi}{3} \int_0^{\tau_0} a^4(\tau)(\tau_0 - \tau)^3 \left[\frac{1}{\kappa} \Gamma - P_{eff}(a) \right] d\tau, \quad (13)$$

where $P_{eff}(a) = P_m - \mathcal{L}_m$ and $\Gamma = (k/a^2)[2/3 + \tau/(\tau_0 - \tau)]$. I_{cl} is defined to be I_{tot} evaluated with $\mathbf{g}_{\mu\nu}$ and the matter fields obeying their classical field equations. Hence,

$$I_{cl} = \frac{4\pi}{3} \int_0^{\tau_0} a^4(\tau)(\tau_0 - \tau)^3 \left[\left(\frac{1}{\kappa} \frac{k}{a^2} \right) \left[\frac{2}{3} + \frac{\tau}{(\tau_0 - \tau)} \right] - P_m - \mathcal{L}_m \right] d\tau, \quad (14)$$

where P_m can be contributed by radiation, dark matter and baryonic matter which are labeled by “rad”, “dm”, and “b” respectively. For the dark matter and baryonic matter $P_{rad} = \rho_{rad}/3$, $\mathcal{L}_{rad}/\rho_{rad} \approx 0$, $P_{dm}/\rho_{dm} \approx 0$ and $\mathcal{L}_{dm}/\rho_{dm} \approx 0$. For baryonic matter, $p_b/\rho_b \approx 0$, $\mathcal{L}_b \approx -\zeta_b \rho_b$, where for some $\zeta_b \sim O(1)$ is calculated by QCD. For the structures of baryonic matter, the chiral bag model gives the estimate $\zeta_b \sim 1/2$ [5]. As a result that $\rho_b \gg \rho_{rad}$, the dominant contribution to P_{eff} comes from baryonic matter and $P_{eff} \approx \zeta_b \rho_b$. The terms in I_{cl} only depends

on λ through the scale factor $a(\tau)$, $\Gamma \propto a^{-2}$ and $P_{eff} \approx \zeta_b \rho_b \propto a^{-3}$. Therefore, equation (13) as well as (14) can be rewritten again as follows:

$$I_{cl} = \frac{4\pi}{3} \int_0^{\tau_0} a^4(\tau)(\tau_0 - \tau)^3 \left[\left(\frac{1}{\kappa} \right) \frac{1}{a^2(\tau)} - \frac{1}{a^3(\tau)} \right] d\tau. \quad (15)$$

The above action can be more simplified:

$$I_{cl} = \frac{4\pi}{3\kappa} \int_0^{\tau_0} (\tau_0 - \tau)^3 [a^2(\tau) - a(\tau)] d\tau. \quad (16)$$

For the matter dominate era, suppose the scale factor $a(\tau) \propto \tau^{2/3}$, then one can derive that

$$I_{cl} = \frac{4\pi}{3\kappa} \int_0^{\tau_0} (\tau_0 - \tau)^3 [\tau^{4/3} - \tau^{2/3}] d\tau \quad (17)$$

$$= \left(\frac{4\pi}{3\kappa} \right) \left(\frac{243\tau_0^{14/3}(11\tau_0^{2/3} - 26)}{80080} \right), \quad (18)$$

and the classical action, I_{cl} , versus observer's cosmic time, τ_0 , is plotted in FIG. 1.

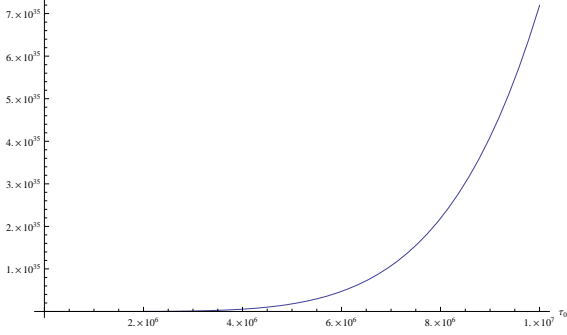


FIG. 1: A I_{cl} - τ_0 diagram which interprets that the monotonic increasing tendency corresponding to observer's cosmic time increasing.

In conformity with [5], section IIA, one has the following form in the classical limit:

$$Z[\mathcal{M}] \approx \sum_{\alpha=1}^N \sum_{\Lambda} \mu[\Lambda] \exp(iI_{cl}[\Lambda; \mathcal{M}]). \quad (19)$$

Motivated by simplicity and extracting the significant theoretical meaning, now assuming that the weight function can be fixed as unit function and only focus on the classical solution, i.e. the Universe we live then the first summation can temporarily neglect. Then one can easily derive the *sui generis* partition:

$$Z[\mathcal{M}] \approx \exp(iI_{cl}[\Lambda; \mathcal{M}]). \quad (20)$$

Now, substitute equation (18) into equation (20), then we could have

$$Z[\mathcal{M}] \approx \exp \left(\left(\frac{4\pi i}{3\kappa} \right) \left(\frac{243\tau_0^{14/3}(11\tau_0^{2/3} - 26)}{80080} \right) \right). \quad (21)$$

this partition function can be plotted by decomposed into real-part, $Re[Z]$, and imaginary part, $Im[Z]$. The parameter is observer's cosmic time, τ_0 as in FIG. 1.

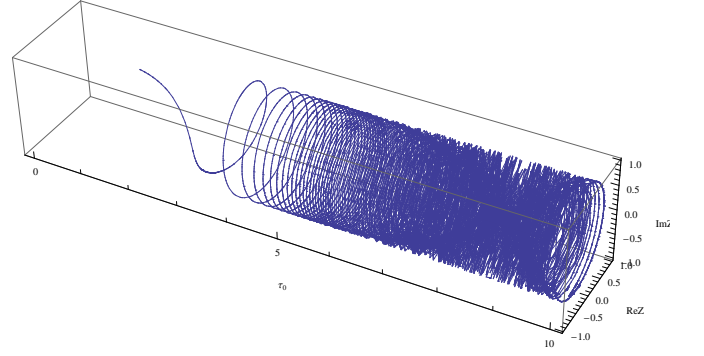


FIG. 2: A 3D diagram for illustrating the rate changing as the observer's cosmic time increasing.

According to FIG. 1; apparently, there is a tendency that as τ_0 increasing, the corresponding value of classical action proliferating as well. This tendency is also reflected in FIG. 2. In the finite temperature field theory perspective, the physical meaning of the observer's cosmic time, τ_0 , is related to inverse temperature with respect to the partition function which is derived as follows. Applying wick rotation, then τ_0 replaced by $-it$. Furthermore, $\beta = t/\hbar = t$, where $\beta := 1/kT$, k is the Boltzmann constant and $\hbar = 1$. Thus we get a vital relation $t = 1/kT = i\tau_0$.

Consider that we can investigate critical phenomena, phase-transitions, of a system by investigating the distribution of Lee-Yang zeros. This notion is first proposed by Lee and Yang in 1952[27, 28], and generalized by Fisher and many other following researchers. Currently 3D quantum gravity(QG) still has many ambiguities[29, 30], especially on partition function[31]. It could be quite helpful to construct a *de facto* QG. E. Witten[32] applied the notion of Lee-Yang zeros to partition function of 3D quantum gravity and found that this procedure can be used to explain Hawking-Page phase transitions of black holes. All Lee-Yang zeros can be located by tessellation mappings. Among many other relevant connections, Hawking-Page phase transitions, deconfinement in QCD phase transitions, also be called quark-gluon-plasma(QGP), is the most significant one. Because Witten's approach is concentrated on the partition function of the BTZ black holes with different geometries, it's natural to think that whether the partition function in Shaw-Barrow's can also have phase transition. If so, it's straightforward to make an investigation on distribution of locations of Lee-Yang zeros. Motivated by this, before one can easily observe the distribution of the locations, a mathematical analysis of the mathematical structure of the partition function in equation (21) is necessary. Thus, the goal in the remaining of this section is concentrated to lay on foundation for investigating the distribution of Lee-Yang zeros of the partition function which is corresponds to the Universe.

Rewritten summation of effective CC, Λ , in equation (19) into observer's cosmic time, since we have al-

ready known that Λ depend on τ in the previous discussion (Λ is in the implicit expression of $a(\tau)$, and $a(\tau) \propto \tau^{2/3}$, thus Λ can be labeled by $\tau = \tau_0$). Change variable $\tau_0 = n, n \in \mathbf{N}$.

$$Z[\mathcal{M}] \approx \lim_{N \rightarrow \infty} \sum_{n=0}^N \exp \left(\left(\frac{4\pi i}{3\kappa} \right) \left(\frac{243n^{14/3}(11n^{2/3} - 26)}{80080} \right) \right), \quad (22)$$

Simplify the above expression,

$$Z[\mathcal{M}] \approx \lim_{N \rightarrow \infty} \sum_{n=0}^N \exp \left(\left(\frac{4\pi i 243}{3\kappa 80080} \right) \left(n^{14/3}(11n^{2/3} - 26) \right) \right). \quad (23)$$

Assume

$$A_1 \equiv \left(\frac{4\pi 243}{3\kappa 80080} \right). \quad (24)$$

Therefore,

$$Z[\mathcal{M}] \approx \lim_{N \rightarrow \infty} \sum_{n=0}^N \exp \left(A_1 i \left(11n^{16/3} - 26n^{14/3} \right) \right). \quad (25)$$

$$\approx \lim_{N \rightarrow \infty} \sum_{n=0}^N \left\{ \exp \left(A_1 i 11n^{16/3} \right) \exp \left(-A_1 i 26n^{14/3} \right) \right\}. \quad (26)$$

Define that $z \equiv (\exp(A_1 i))^{1/3}$. Thus, the equation (26) is taking the following form:

$$Z[\mathcal{M}] \approx \lim_{N \rightarrow \infty} \sum_{n=0}^N \left(z^{11n^{16}} z^{-26n^{14}} \right) \quad (27)$$

$$= \lim_{N \rightarrow \infty} \sum_{n=0}^N \left(z^{11n^{16} - 26n^{14}} \right) \quad (28)$$

$$= \sum_{K=0}^{\infty} z^K, \quad (29)$$

where I let $(11n^{16} - 26n^{14}) := K, K \in \mathbf{Z}$.

Finally, according to equation (29) we derive:

$$Z[\mathcal{M}] = \sum_{K=0}^{\infty} z^K \quad (30)$$

$$= \prod_{i=1}^{\infty} \left(1 - \frac{z}{z_i} \right), \quad (31)$$

where the fundamental theorem of algebra have been applied, and z_i denotes the partition function zeros(after Lee-Yang theorem be applied, we can call them as Lee-Yang zeros).

The point is that the equation be derived in equation (30) and (31) imply that Lee-Yang theorem can be applied in this theory. Furthermore, this analysis method can be generalized to whole classes of Shaw-Barrow theory. Thus, phase-transitions can be numerically analyzed by using equation (30) and (31) with keeping track of the location and distribution of Lee-Yang zeros.

V. CONCLUSION

In summary, this note pointed out two problems in [1] in section IV-A; furthermore, a possible way-out is discussed in section IV-B, and a meaningful direction and method is given in section IV-C. Besides, there are at least two important open questions should be focused on: First, to make a further investigation on cosmic holography, and to establish a concrete understanding of the question that under what precise conditions this principle can be applied? Secondly a further numerical analysis of the distribution of Lee-Yang zeros are necessary to be constructed. In fact, a systematic description of the distribution of Lee-Yang zeros are still missing.

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